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A MONTE CARLO SIMULATION OF GUIDANCE ACCURACY EVALUATION

by

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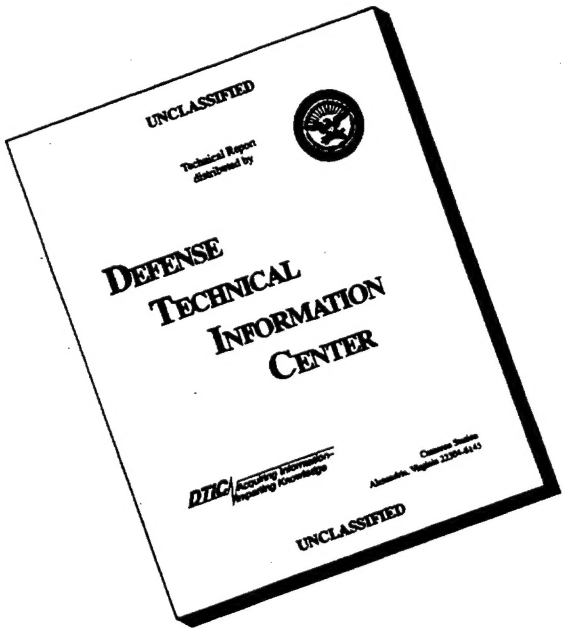

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A Monte Carlo Simulation of Guidance Accuracy Evaluation

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Abstract: It is necessary to carry out a Monte Carlo simulation of guidance accuracy evaluation in order to solve the problem of evaluating guidance accuracy because of the few strategic missile flight tests that have been carried out. A Monte Carlo simulation method of strategic missile guidance accuracy evaluation, as well as several important problems involved in engineering application and specific methods to solve these problems are introduced.

Key Words: Strategic missile, guidance error, Monte Carlo method, mathematical model

1. Introduction

Strategic missile accuracy is a vitally important tactical and technical index. To evaluate the guidance accuracy, the conventional method is to conduct a statistical evaluation of the missile falling point during a flight test, which requires a certain number of flight tests. In reality, however, the number of missile flight tests is extremely limited due to their very high costs and the restrictions imposed by various conditions at test sites. Therefore, a strategic missile flight test essentially bears the character of accuracy verification. To overcome the difficulties confronted by the conventional method in this area, a Monte Carlo simulation of strategic missile guidance accuracy evaluation has been proposed and is regarded as a significant technical approach in solving this problem.

The Monte Carlo method is an experimental mathematical method with which a statistical check can be carried out by using random numbers, and the characteristic statistical values calculated (such as mean value, variance, probability, etc.) are taken as a numerical solution to a particular problem to be solved. More specifically, this is a method of reconstructing the physical process of a practical problem under research by using a mathematical probability model and calculations of the characteristic statistical values of that process. With this method, the model established can be tested many times, and based on this procedure, the test data are processed statistically so as to derive the characteristics of the foregoing process as the statistically estimated values of the process parameters. In accordance with the amount of scattering of these parameters, the degree of approximation in solving the problem can be determined in the sense of probability.

The Monte Carlo method itself is well developed. With the development of computers in the past few decades, this method has been applied more and more widely in various scientific and technological engineering areas, and has solved a large number of practical problems. This paper mainly discusses the Monte Carlo simulation of strategic missile guidance evaluation, as well as some major problems that this kind of simulation encounters in its engineering applications and their specific solutions.

2. Basic Aspects of Monte Carlo Simulation of Guidance Accuracy Evaluation

Simulating strategic missile guidance accuracy with the Monte Carlo method proves to be an extremely complicated numerical simulation test of engineering systems. It is supposed to be accomplished in three steps, namely building a mathematical model, conducting a simulation test and analyzing the simulated

test data. This procedure involves eight major aspects listed as follows:

a) Analysis and research on various sources of errors affecting the guidance system accuracy, as well as collection and accumulation of the number of errors;

b) Constructing probability models for guidance tool errors and guidance method errors;

c) Constructing a mathematical model of the trajectory which is used to describe the actual missile flight;

d) Confirmation of the correctness of the simulation model established;

e) Selection of a proper random variable sampling method;

f) Design of the simulation test which includes correct generation and statistical check of pseudorandom numbers, design of a method of minimizing the simulation test variance, and determination of simulation test times required for a given accuracy;

g) Analysis and processing of simulation test data, and

h) Computing software design, debugging and verification on a computer which is necessary for implementing the Monte Carlo simulation test.

3. Method of Constructing the Mathematical Model

A key issue in evaluating guidance accuracy with Monte Carlo simulation is how to correctly establish the mathematical model. Construction of the mathematical model involves two major aspects:

a) Constructing a set of coupled differential equations which can precisely describe the actual missile flight movement, i.e. a mathematical model of the trajectory;

b) Fully taking advantage of the information from existing ground tests and flight tests so as to collect and accumulate various statistical data concerning error levels and build probability models for differing error levels.

Among other things, the construction of probability models is the most difficult and most important task in establishing the mathematical model. The probability models constructed are required not only to be able to reflect the reality of the object simulated, but also to be simple and easy to realize so that the solution coincides with the probability distribution or mathematical expectation of the model constructed. Since a simulation test without a certain confidence level makes no sense, it is necessary to confirm the correctness of the simulation model constructed before the simulation test can be started.

Based on an analysis of the various levels of errors which affect guidance accuracy, two methods of building error level probability models are introduced.

3.1 Analysis of Various Error Levels Affecting Guidance Accuracy

A ballistic missile system may have two kinds of error sources: guidance errors and non-guidance errors. The inertial guidance system consists of an attitude reference, an accelerometer, a computer and a control system. The guidance errors can be further subdivided into guidance tool errors and guidance method errors.

The problems which affect the guidance tool errors include: initial alignment errors (leveling, pointing) of a stable platform before the launch; errors in missile velocity measurement with the accelerometer; missile attitude measurement error caused by gyrodrift; in-missile computer errors, etc.

The problems which affect the guidance method errors include: missile launch quality deviation; deviation in engine propellant agent per-second consumption; engine thrust ratio

deviation; engine thrust line inclination; deviation caused by wind interference and atmospheric parameter changes, etc.

In terms of kinematic trajectory equation calculations, the foregoing problems can be subdivided into three categories as follows:

a) There is no exact knowledge about the number of constants in the kinematic equation, such as take-off quality, engine thrust line inclination;

b) The actual parameter change regularities do not conform to the design, such as engine propellant agent per-second consumption deviation, variation of atmospheric parameters with altitude, etc.;

c) Parameters which are not included in the kinematic equation, such as inertial instrument errors, non-uniformity of wind and atmospheric parameters, etc.

Any of the foregoing problems has a random character, i.e. their effect on any particular in-flight missile cannot be predicted in advance. Every error, although it has an uncertain value, abides by a given statistical distribution and therefore its effect on guidance errors can be calculated as long as its distribution is known.

3.2 Method of Constructing a Probability Model

When the original data on various errors which affect the guidance accuracy are gathered and properly processed, a set of observation data x_1, $x_2 \ldots x_N$ which reflect a particular random process of a particular error can be acquired. If these observation data are independent of each other, the model can be constructed by selecting the random variable probability distribution, or otherwise if they are dependent, correlated to

each other; the model should be built with the time sequence analysis method.

3.2.1 Model Construction Based on Selection of Random Variable Probability Distribution

As mentioned above, when the observation data which reflect a random variable of a given error are independent, a random variable probability distribution can be selected to construct a probability model. To achieve this goal, the usual practice is to select a proper probability distribution type before carrying out a parameter estimation.

3.2.1.1 Method of Selecting Probability Distribution Type

Based on a priori knowledge including an understanding of the particular random variable and data possessed, it can be decided which distribution should be adopted or which distribution should be rejected. In this case, data play a decisive role in selecting the probability distribution. With adequate data, it is possible to select a probability distribution which conforms with the actual situation of the system, and also to check the correctness of the probability distribution selected.

The following description is given for some specific methods which can be used in selecting an appropriate probability distribution type with given data:

3.2.1.1.1 Histogram Method

A histogram is a graphic description of data evaluation distribution, and also an approximation of the random variable density function. By estimating the shape of a density curve with a histogram, it can be roughly seen which type of probability distribution it looks like.

3.2.1.1.2 Probability Diagram Method

The probability diagram method is used to estimate the theoretical distribution function directly with the empirical distribution function $F_N(x)$. The method generally used in engineering is the probability paper method. With the probability paper method, some theoretical distributions can be put in a given coordinate in straight lines, and in the meantime, an empirical distribution function curve is also plotted in the same coordinate system for a comparison to find out their resemblance and difference. For instance, the probability diagram of a particular datum can be drawn in a normal state probability paper, and if the datum is distributed in a straight line, it is considered to abide by the normal state distribution. In this case, many kinds of probability papers can be prepared in advance, including a normal state probability paper, an algorithmic normal probability paper, a Weber probability paper, etc.

3.2.1.1.3 Point Estimation Method

With this method, sample variance S^2 and sample mean value can be calculated in accordance with a given sample datum and further, the estimated value $\bar{x}$ of the variance coefficient $\hat{\delta}_c$ can be calculated on the basis of the following equation

$$\hat{\delta}_c = \frac{S}{\bar{x}} \qquad (1)$$

There are different variance coefficient δ_c evaluation ranges in response to different probability distributions, which can be found in a table. It can be judged then, based on the $\hat{\delta}_c$ value calculated, which probability distribution is obeyed by the foregoing sample datum.

3.2.1.2. Parameter Estimation Method

After a probability distribution type is selected, the parameters of that distribution should be estimated. The commonly used methods of estimating such parameters are the moment estimation method and the maximum similarity estimation method.

3.2.1.2.1 The Moment Estimation Method

Since x_1, $x_2 \ldots x_N$ are known as a sample of the random variable X, the sample k-order original point moment can be used as the estimation of the overall k-order original point moment as shown in the following equation

$$\hat{\mu}_k = \frac{1}{N}\sum_{i=1}^{N} x_i^k \tag{2}$$

For instance, it is known that the random variable $X \sim N(\mu, \sigma^2)$, the moment estimated amounts of μ and σ^2 are

$$\left.\begin{aligned} \hat{\mu} &= \hat{\mu}_1 = \frac{1}{N}\sum_{i=1}^{N} x_i = \bar{x} \\ \hat{\sigma}^2 &= \hat{\mu}_2 - \hat{\mu}_1^2 = \frac{1}{N}\sum_{i=1}^{N} (x_i - \bar{x})^2 \end{aligned}\right\} \tag{3}$$

3.2.1.2.2 The Maximum Similarity Estimation Method

The basic idea of the maximum similarity estimation method signifies that if the maximum value of the similarity function $L_N(x_1, x_2 \ldots x_N, \theta_1, \theta_2 \ldots \theta_M)$ is selected at $\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_M$ (i.e. the probability of their generation is the maximum parameters $\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_M$), then $\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_M$ can be

taken as the maximum similarity estimated value of the overall parameter θ_1, θ_2,...,θ_M.

As for the normal state distribution, its maximum estimated similarity amount is identical to the moment estimated amount, while other distributions can be directly calculated by using many maximum similarity estimation formulas for common probability distribution parameters, which are listed in statistics books, as is done in practical computations.

3.2.2 Model Construction Based on Time Series Analysis Method

3.2.2.1. The Time Series Analysis Method

During quantitative measurement of one variable or a group of variables X(t) derived from a production or research process, a discrete digitalized ordered number ensemble $x(t_1),x(t_2)...x(t_N)$ with time t as an independent variable can be obtained at the moment $t_1<t_2<...<t_N$, which is generally referred to as a digital time series. In practical problems, the independent variable t represents not only time, but also various physical implications, including length, temperature, velocity or other physical quantities of monotonically increasing evaluation.

Generally speaking, it is extremely difficult to express a time series with a completely certain function or function group. In most cases, its statistical law is displayed, and therefore its evaluation law can be described statistically through a probability distribution function or function group. This kind of time series is referred to as a random time series, or a random process, as it is called in the probability theory. Most time series can be regarded as a reality, or a sample function of the random process.

If time series data are not independent but correlated to each other, this correlation must be taken into account in model construction, i.e. model construction should be based on the time series analysis method instead of the probability distribution selection method.

The time series analysis method is a statistical mathematical method, which is used for time series analysis and research. The major time series analysis methods are all based on the assumption that data samples are all derived from a smooth random process, i.e. their statistical characteristics such as mean values, variances and correlation functions do not change with the passage of the original time point. In reality, however, most random processes are not smooth, and therefore, the non-smooth time series must initially be smoothed.

The parameter method and the difference method are commonly used as smoothing methods for non-smooth time series. In this paper, our discussion is devoted to the parameter method alone.

The non-smooth time series $x(t)$ is usually described with the following addition model (other non-addition models can be properly converted to addition models):

$$x(t)=d(t)+\eta(t) \tag{4}$$

where $\eta(t)$ is assumed to be a time series; $d(t)$ is a function term indicating the time series $x(t)$ with an apparent tendency, which is presumably formed through superimposition of the following two functions:

$$d(t)=f(t)+p(t) \tag{5}$$

where $f(t)$ is a main value function term indicating a long-term change tendency of the time series; $p(t)$ is a periodic function term which represents some periodic change factors affecting the

time series. Their identification and extraction methods [3] are briefly introduced in the following section.

3.2.2.1.1 Method of Identifying and Extracting Main Value Function Term f(t)

The following functions

$$f(t)=\alpha_0+\alpha_1 t+\alpha_2 t^2+\alpha_3 t^3+\alpha_4 t^4+\alpha_5 t^{-1}+\alpha_6 t^{-2}+\alpha_7 t^{\frac{1}{2}}+\alpha_8 t^{-\frac{1}{2}}+\alpha_9 e^{-t}+\alpha_{10}\ln t$$

are used as an approximation of the actual main value function term so as to fit into the given time series x(t). In other words, in the the method of least squares, a computer is used to identify and select the function f(t), estimate the regression coefficient α_i (i=0, 1, 2...10), and finally give the form of the main value function term f(t) by using the gradual regression analysis method and under the checked values F_1 and F_2 of the given round-off forecast factor F. If the estimated regression coefficient α_i is zero in any case, it can be accepted then that there is no main value function term f(t).

3.2.2.1.2 Method of Identifying and Extracting Periodic Function Term p(t)

When the main value function term f(t) is removed from the given time series x(t), an ordered number ensemble is obtained as

$$x_1(t)=x(t)-f(t)$$

Consider the periodic series model

$$x_1(t)=\alpha_0+\sum_{i=1}^{l}(\alpha_i\cos\omega_i t+\beta_i\sin\omega_i t)+\eta(t)$$

where α_0, α_i, β_i, ω_i i=1,2,...,l) and l are all parameters to be identified and estimated.

Now, the periodic function term p(t) can be identified and

extracted with the method of periodic diagram analysis in the least square method.

After the calculated function term d(t) is removed from a given non-smooth time series x(t), the time series $\eta(t)$ can be derived as

$$\eta(t)=x(t)-d(t)=x(t)-f(t)-p(t) \qquad (6)$$

Obviously, $\eta(t)$ is a smooth time series.

3.2.2.2 Construction of Smooth Linear Model

As soon as the smooth time series $\eta(t)$ is derived, it is necessary to determine what kind of smooth linear model should be adopted through model identification method. The commonly used smooth linear models are the self-regression model AR(p), sliding average model MA(q) and self-regression-sliding average model ARMA(p, q).

The time series model is established in the order of model identification, its type and order number determination and its parameter estimation.

The AR(p) model parameters are estimated with a linear estimation method. In this case, the least square method can be used to derive the estimated model parameters derived from a solution of the standard equation. As for the parameters of the MA(q) and ARMA(p, q) models, their estimation appears much more complex and requires the application of the non-linear least square method.

It is to be noted that the foregoing time series analysis methods of model construction are applicable only in the case

when the data concerned obey the normal state of distribution, while the input process of an actual system is often a non-normal state process. Thereby, data should be converted from a non-normal distribution to a normal distribution before the foregoing methods can be used. Specifically, the conversion methods include the algorithmic conversion method and the normal conversion method based on the empirical distribution function [2].

4. Random Number Generation and Verification Methods

The foundation of Monte Carlo simulation is that a digital computer can rapidly generate random numbers with ideal statistical properties, which are evenly distributed in the [0, 1] interval. With random numbers, the random number series with different distribution regularities required from the simulation can be derived through conversion, sampling, etc. The quality of random series has a direct effect on the confidence level of the simulation result.

Generally, random numbers are generated by using mathematical methods, such as the multiplication congruence method, addition congruence method, mixed congruence method and quadratic method. Procedures are readily available for all these methods to be selected.

As for the pseudorandom series generated on a computer with mathematical methods, they have no real random character. These pseudorandom numbers must be checked statistically before being used as required random numbers, so that there will not be any large systematic error in the Monte Carlo simulation result.

There are many random number statistical verification methods, which include parameter checking, uniformity checking, independence checking, continuity checking and combination

regular checking, etc. It is necessary to select, based on the specific problem simulated, an appropriate verification method, and determine its proper marked level and rejection domain.

5. Falling Point Accuracy Evaluation Method

Suppose n Monte Carlo simulation tests have been conducted, and the observation values of falling point deviation caused by a guidance error have been calculated as

$$\lambda_1,\ \lambda_2,\ \cdots,\ \lambda_n$$

then the falling point accuracy and accuracy evaluation can be carried out over the values of that sample group as described in the following section.

5.1 Sample Characteristic Value Calculations

Sample mean value

Sample variance

Sample standard difference

$$\left.\begin{aligned} \bar{\lambda} &= \frac{1}{n}\sum_{i=1}^{n}\lambda_i \\ S^2 &= \frac{1}{n}\sum_{i=1}^{n}(\lambda_i-\bar{\lambda})^2 \\ S &= \sqrt{S^2} \end{aligned}\right\} \qquad (7)$$

5.2 Estimation of Confidence Interval between Mean Value and Variance With a Given Confidence Level

Generally, missile falling point deviation obeys the normal state distribution (μ, σ^2). The following discussion contributes to the estimation of the confidence interval of μ, σ^2.

5.2.1 Estimation of Mean Value Confidence Interval

5.2.1.1 With a Known Variance ($\sigma^2=\sigma_0^2$)

Under a given confidence level (1-α), the confidence interval of mean value μ is:

$$\left(\bar{x} - u_{\alpha} \cdot \frac{\sigma_0}{\sqrt{n}} \ , \ \bar{x} + u_{\alpha} \cdot \frac{\sigma_0}{\sqrt{n}}\right) \tag{8}$$

where n--sample capacity;

$\bar{x}$ sample mean value;

u_{α}--α--horizontal bilateral fractile of standard normal state distribution, which is listed in the related table.

5.2.1.2 With an Unknown Variance

With a given confidence level (1-α), the confidence interval of mean value μ is:

$$\left(\bar{x} - t_{\alpha} \cdot \frac{S}{\sqrt{n}} \ , \ \bar{x} + t_{\alpha} \cdot \frac{S}{\sqrt{n}}\right) \tag{9}$$

where t_{α} is α--horizontal bilateral azimuth number of t distribution with degree of freedom (n-1).

5.2.2 Estimation of Variance Confidence Interval

5.2.2.1 With a Known Mean Value μ, ($\mu=\mu_0$)

With a given confidence level (1-α), the confidence interval of variance σ^2 is:

$$\left(\frac{1}{x^2_{\frac{\alpha}{2},n}} \sum_{i=1}^{n} (x_i - \mu_0)^2, \ \frac{1}{x^2_{1-\frac{\alpha}{2},n}} \sum_{i=1}^{n} (x_i - \mu_0)^2\right) \tag{10}$$

where $x^2_{q,n}$ is q--horizontal upper side fractile of x^2 distribution with a degree of freedom n; q respectively represents α/2 and 1-α/2.

5.2.2.2 With an Unknown Mean Value μ

With a given confidence level (1-α), the confidence interval of variance σ^2 is:

$$\left(\frac{(n-1)S^2}{x^2_{\frac{\alpha}{2},\,n-1}} \;,\; \frac{(n-1)S^2}{x^2_{1-\frac{\alpha}{2},\,n-1}} \right) \tag{11}$$

6. Method of Determining Simulation Test Times Required from a Given Accuracy

Since Monte Carlo simulation for strategic missile guidance accuracy evaluation requires a host of tests, and the trajectory calculations are time-consuming, a heavy volume of computer time has to be spent to ensure high precision simulation calculations. Under such a scenario, research should be carried out on how to determine the minimum simulation test times n_A without sacrificing the required simulation accuracy in solving specific engineering problems.

The following equation reflects the relationships among the confidence level (1-α) of the estimated value, the confidence interval 2 calculated and the simulation times n realized.

$$P\{|\hat{x} - x| < \varepsilon\} = 1 - \alpha \tag{12}$$

A formula is derived from Eq. (12) which indicates that the test times n_A needed for ensuring a given accuracy r_A in simulation calculations can be derived as follows:

$$n_A = \min\left\{ j \geq n : \frac{S}{\sqrt{n}} t_\alpha \leq r_A \right\} \tag{13}$$

where $\frac{S}{\sqrt{n}} t_\alpha$ is the half length of mean value μ confidence interval under an unknown variance σ^2, i.e. ε;

r_A is an absolute accuracy of the confidence interval,

which is defined as the half length of the given confidence interval;

t_α is α--horizontal bilateral fractile of t distribution with degree of freedom (n-1), which can be found in the related table.

7. Method of Minimizing Test Variance

7.1 Significance of Minimizing Test Variance

When the Monte Carlo method is used to carry out simulation calculations, its method error ε can be written as

$$\varepsilon = \frac{\sigma}{\sqrt{n}} t_\alpha \tag{14}$$

where ε is the given simulation error;

σ is the standard difference of the random variable selected;

n is test times.

Equation (14) shows that there are two approaches to increase simulation accuracy with the given confidence level $(1-\alpha)$: one is to increase test times n, while the other is to minimize variance σ^2.

As ε is directly proportional to $\frac{1}{\sqrt{n}}$, a 10-fold increase of simulation accuracy requires a 100-fold increase in test times, which inevitably results in a considerable increase in computer time. On the other hand, since ε is in direct proportion to σ, minimizing variance σ^2 will bring about a striking positive result. Theory and practice suggest that a combination of various variance minimization methods and skills can raise the simulation accuracy by 10 or even 100 times.

7.2 Methods of Minimizing Test Variance

There are many methods which can be used to minimize test variance. Additionally, different methods can be used for the same given problem. Here the skill of minimizing test variance, the dual variable method, is introduced; it is well-suited to engineering problems.

The major idea of the dual variable method is as follows: Suppose x_f and y_f are determined from distribution f(x, y) through sampling, with a mathematical expectation of $\theta(x_f, y_f)$. x'_f and y'_f are also determined from the same distribution f(x, y) through sampling, whose mathematical expectation is $\theta(x_f, y'_f)$. If $\theta(x_f, y_f)$ and $\theta(x'_f, y'_f)$ are in negative correlation, this pair of estimations with reverse variation but mutual compensation are referred to as a dual variable. Therefore,

$$\hat{\theta} = \frac{1}{2}\left[\theta(x_f, y_f) + \theta\left(x'_f, y'_f\right)\right]$$

can be taken as the estimated value of θ, which is a non-deviated estimation. The variance of the estimated value $\hat{\theta}$ is

$$\frac{1+\rho}{2}\sigma^2\{\theta(x, y)|f(x, y)\}$$

When the correlation coefficient ρ=-1, the variance of the estimated value $\hat{\theta}$ as a dual variable sampling skill is equal to zero.

Thus, the dual variable method is virtually: to find one (or several) random variable(s) with the same mathematical expectation value as that of the original random variable (the mathematical expectation value of this random variable is the quantity to be calculated), and try as much as possible to place them in negative correlation. Based on this, the linear combination of the two (or more) random variables is taken as the

estimated value. By using this method, the test variance can be minimized remarkably.

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